


# Parallel Techniques

- Embarrassingly Parallel Computations
- Partitioning and Divide-and-Conquer Strategies
- Pipelined Computations
- Synchronous Computations
- Asynchronous Computations 
- Load Balancing and Termination Detection

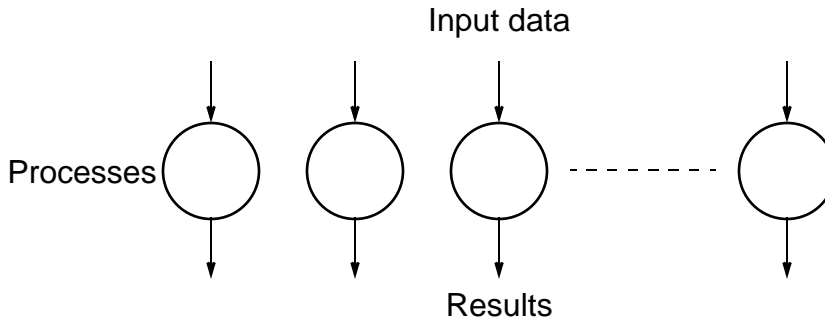
Not covered in 1st edition  
of textbook

## Chapter 3

# Embarrassingly Parallel Computations

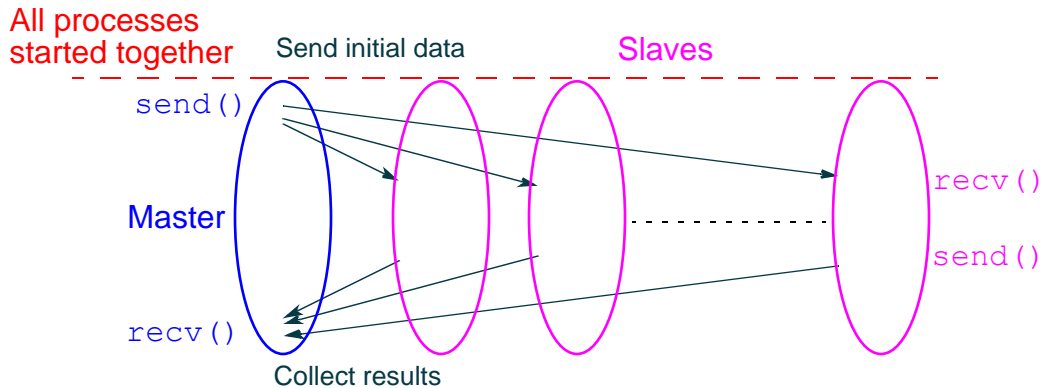
# Embarrassingly Parallel Computations

A computation that can **obviously** be divided into a number of completely independent parts, each of which can be executed by a separate process(or).



No communication or very little communication between processes  
Each process can do its tasks without any interaction with other processes

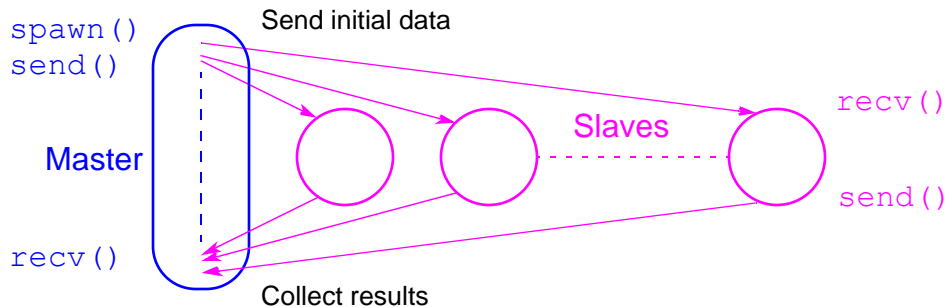
# Practical embarrassingly parallel computation with static process creation and master-slave approach



MPI approach

# Practical embarrassingly parallel computation with dynamic process creation and master-slave approach

Start Master initially



PVM approach

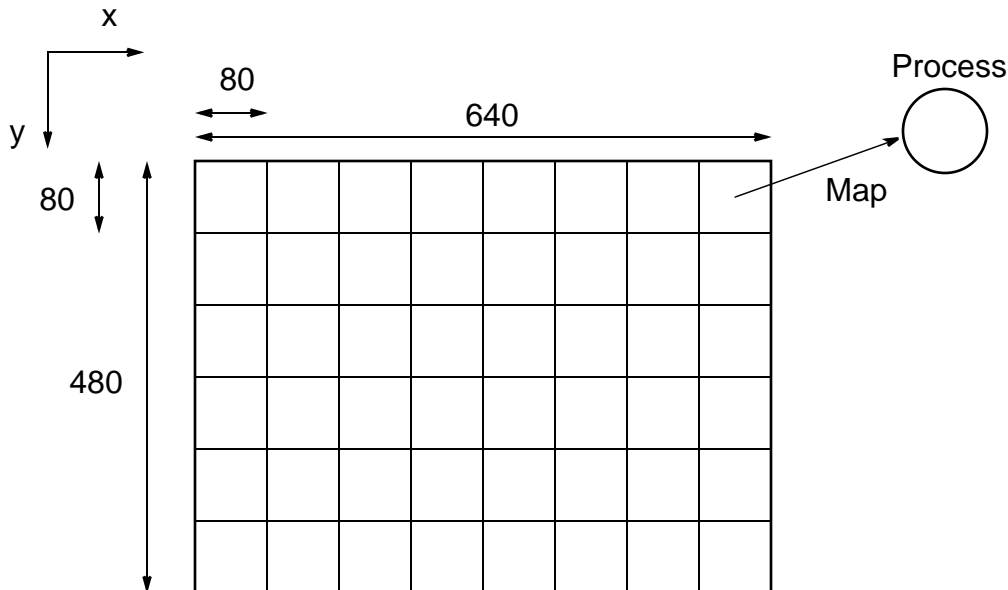
# Embarrassingly Parallel Computation Examples

- Low level image processing
- Mandelbrot set
- Monte Carlo Calculations

## Low level image processing

Many low level image processing operations only involve local data with very limited if any communication between areas of interest.

# Partitioning into regions for individual processes.



Square region for each process (can also use strips)



# Some geometrical operations

## Shifting

Object shifted by  $x$  in the  $x$ -dimension and  $y$  in the  $y$ -dimension:

$$x = x + \Delta x$$

$$y = y + \Delta y$$

where  $x$  and  $y$  are the original and  $x$  and  $y$  are the new coordinates.

## Scaling

Object scaled by a factor  $S_x$  in  $x$ -direction and  $S_y$  in  $y$ -direction:

$$x = xS_x$$

$$y = yS_y$$

## Rotation

Object rotated through an angle  $\theta$  about the origin of the coordinate system:

$$x = x \cos \theta - y \sin \theta$$

$$y = x \sin \theta + y \cos \theta$$

# Mandelbrot Set

Set of points in a complex plane that are quasi-stable (will increase and decrease, but not exceed some limit) when computed by iterating the function

$$z_{k+1} = z_k^2 + c$$

where  $z_{k+1}$  is the  $(k + 1)$ th iteration of the complex number  $z = a + bi$  and  $c$  is a complex number giving position of point in the complex plane. The initial value for  $z$  is zero.

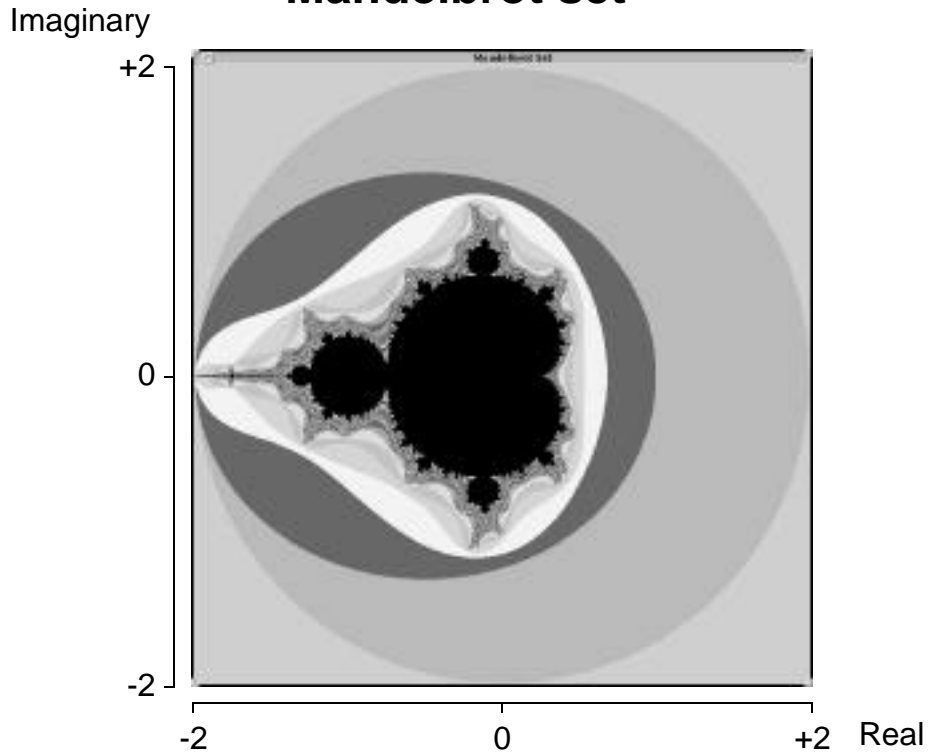
Iterations continued until magnitude of  $z$  is greater than 2 or number of iterations reaches arbitrary limit. Magnitude of  $z$  is the length of the vector given by

$$z_{\text{length}} = \sqrt{a^2 + b^2}$$

## Sequential routine computing value of one point returning number of iterations

```
structure complex {
    float real;
    float imag;
};
int cal_pixel(complex c)
{
    int count, max;
    complex z;
    float temp, lengthsq;
    max = 256;
    z.real = 0; z.imag = 0;
    count = 0;                                /* number of iterations */
    do {
        temp = z.real * z.real - z.imag * z.imag + c.real;
        z.imag = 2 * z.real * z.imag + c.imag;
        z.real = temp;
        lengthsq = z.real * z.real + z.imag * z.imag;
        count++;
    } while ((lengthsq < 4.0) && (count < max));
    return count;
}
```

# Mandelbrot set



# Parallelizing Mandelbrot Set Computation

## Static Task Assignment

Simply divide the region in to fixed number of parts, each computed by a separate processor.

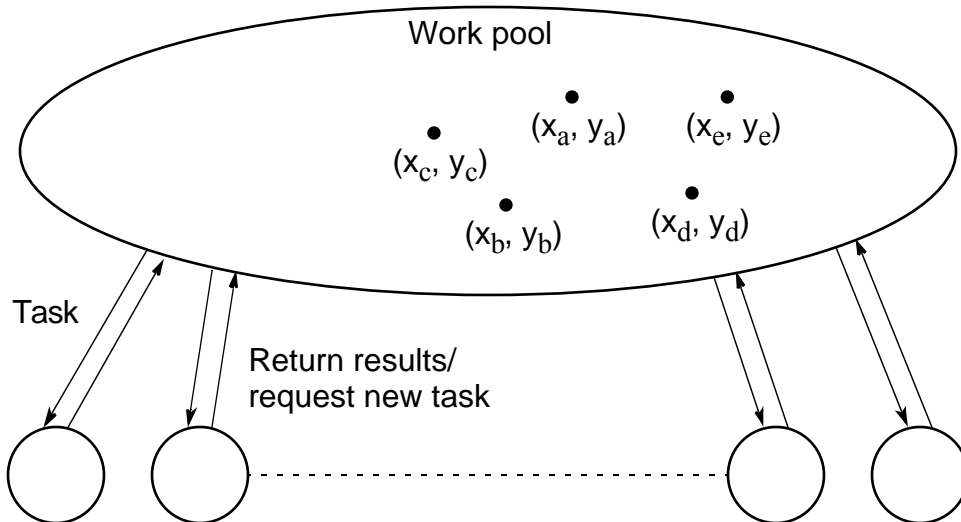
Not very successful because different regions require different numbers of iterations and time.

## Dynamic Task Assignment

Have processor request regions after computing previous regions

# Dynamic Task Assignment

## Work Pool/Processor Farms



## Monte Carlo Methods

Another embarrassingly parallel computation.

Monte Carlo methods use of random selections.

## Example - To calculate

Circle formed within a square, with unit radius so that square has sides  $2 \times 2$ . Ratio of the area of the circle to the square given by

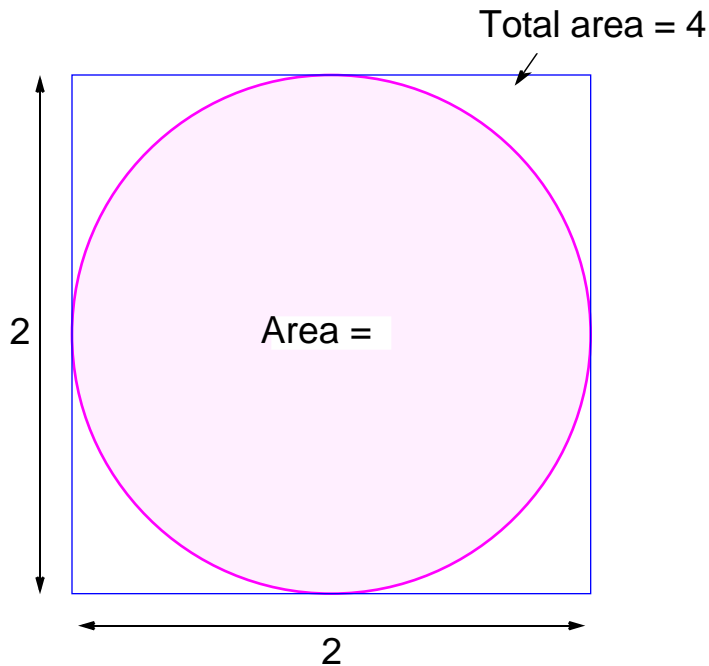
$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{(1)^2}{2 \times 2} = \frac{1}{4}$$

Points within square chosen randomly.

Score kept of how many points happen to lie within circle.

Fraction of points within the circle will be  $1/4$ , given a sufficient number of randomly selected samples.





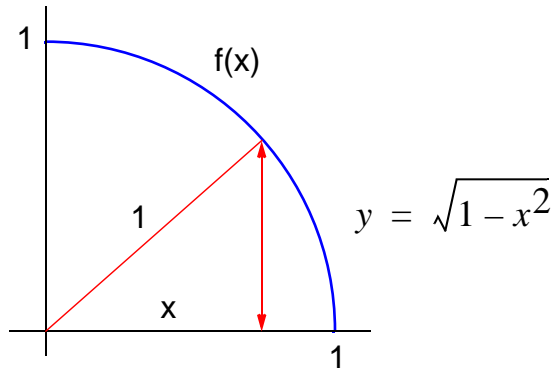
# Computing an Integral

One quadrant of the construction can be described by integral

$$\int_0^1 \sqrt{1-x^2} dx = \frac{\pi}{4}$$

Random pairs of numbers,  $(x_r, y_r)$  generated, each between 0 and 1.

Counted as in circle if  $y_r \leq \sqrt{1-x_r^2}$ ; that is,  $y_r^2 + x_r^2 \leq 1$ .



## Alternative (better) Method

Use random values of  $x$  to compute  $f(x)$  and sum values of  $f(x)$ :

$$\text{Area} = \int_{x_1}^{x_2} f(x) dx = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N f(x_r) (x_2 - x_1)$$

where  $x_r$  are randomly generated values of  $x$  between  $x_1$  and  $x_2$ .

Monte Carlo method very useful if the function cannot be integrated numerically (maybe having a large number of variables)

# Example

Computing the integral

$$I = \int_{x_1}^{x_2} (x^2 - 3x) dx$$

## Sequential Code

```
sum = 0;
for (i = 0; i < N; i++) { /* N random samples */
    xr = rand_v(x1, x2); /* generate next random value */
    sum = sum + xr * xr - 3 * xr; /* compute f(xr) */
}
area = (sum / N) * (x2 - x1);
```

Routine randv(x1, x2) returns a pseudorandom number between x1 and x2.

*For parallelizing Monte Carlo code, must address best way to generate random numbers in parallel - see textbook*

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